

Negative mass hydrodynamics in a spin-orbit-coupled Bose-Einstein condensate

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Negative mass can be realized in quantum systems by engineering the dispersion relation. A powerful method is provided by spin-orbit coupling, which is currently at the center of intense research efforts. Here we measure an expanding spin-orbit coupled Bose-Einstein condensate whose dispersion features a region of negative mass. We observe a range of dynamical phenomena, including the breaking of parity and of Galilean covariance, dynamical instabilities, and self-trapping. The experimental findings are reproduced by a single-band Gross-Pitaevskii simulation, demonstrating that the emerging features – shockwaves, soliton trains, self-trapping, etc. – originate from a modified dispersion. Our work also sheds new light on related phenomena in optical lattices, where the underlying periodic structure often complicates their interpretation.

Newton's laws dictate that objects accelerate in proportion to the applied force. An object's mass is generally positive, and the acceleration is thus in the same direction as the force. In some systems, however, one finds that objects can accelerate *against* the applied force, realizing a *negative* effective mass related to a negative curvature of the underlying dispersion relation. Dispersions with negative curvature are playing an increasingly important role in quantum hydrodynamics, fluid dynamics, and optics [1–8]. Superfluid Bose-Einstein condensates (BECs) provide a particularly lucrative playground to investigate this effect, due to their high reproducibility, tunability, and parametric control. In this letter we report on the experimental observation of negative mass dynamics in a spin-orbit coupled (SOC) BEC. Modeling the experiments with a single-band Gross-Pitaevskii (GP) approach, we clarify the underlying role of the dispersion relation.

We engineer a dispersion by exploiting Raman dressing techniques that lead to a SOC BEC [9–18]. The presence of the Raman coupling, which acts as an effective perpendicular Zeeman field, opens a gap at the crossing of two separated parabolic free-particle dispersion curves. For suitable parameters, the lower dispersion branch acquires a double-well structure as a function of quasimomentum with a region of negative curvature (see Fig. 1a). In our experiments, the BEC is initially spatially well confined, then allowed to expand in one dimension in the presence of 1D spin-orbit coupling. We observe exceptionally rich dynamics, including the breaking of Galilean covariance, which directly manifests as an anisotropic expansion of the symmetric initial state (see Fig. 1b). As their quasimomentum increases, atoms on one side enter the negative mass regime and slow down, demonstrating a self-trapping effect. Related to the onset of the negative effective mass are a limiting group velocity and a dynamical instability [19–21] that leads to the formation of shockwaves and solitons.

Our results suggest that a modified dispersion and the

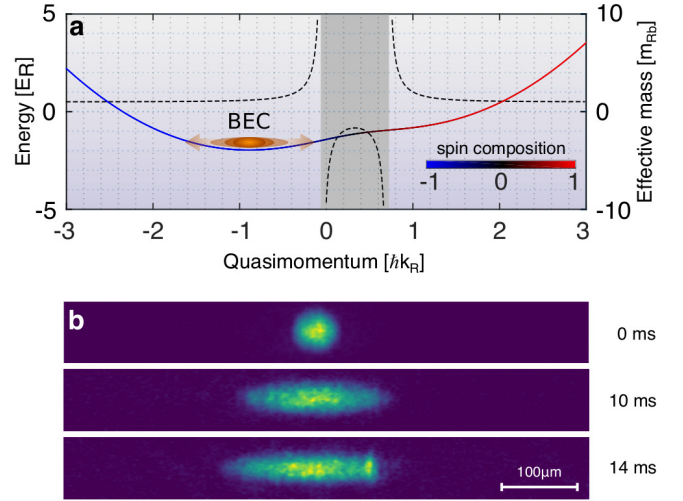


FIG. 1. **a**, Schematic representation of the 1D expansion of a SOC BEC. The asymmetry of the dispersion relation (solid curve) causes an asymmetric expansion of the condensate due to the variation of the effective mass. The dashed lines indicate the effective mass, and the shaded area indicates the region of negative effective mass. The parameters used for calculating the dispersion are $\Omega = 2.5E_R$ and $\delta = 1.36E_R$. The color gradient in the dispersion shows the spin polarization of the state. **b**, Experimental ToF images of the effectively 1D expanding SOC BEC for expansion times of 0, 10 and 14 ms.

corresponding negative effective mass also underlie various “self-trapping” effects seen in optical lattice experiments [22–26]. However, in optical lattices, the origin of the self-trapping effect has been the subject of some controversy due to the occurrence of the underlying potential. This complication is absent on our work.

We start with a BEC of approximately 10^5 ^{87}Rb atoms confined in a cigar-shaped trap oriented along the x -axis of a far-detuned crossed dipole trap (see [27] for details). A spin-orbit coupling is induced along the x -axis by two Raman laser beams that coherently couple atoms in the

$|F, m_F\rangle = |1, -1\rangle$ and $|1, 0\rangle$ states. A quadratic Zeeman shift effectively decouples the $|1, +1\rangle$ state from the Raman dressing, resulting in a system with pseudo-spin $\frac{1}{2}$ where we identify $|\uparrow\rangle = |1, -1\rangle$ and $|\downarrow\rangle = |1, 0\rangle$. In this system, energy and momentum are characterized in units of $E_R = \hbar^2 k_R^2 / 2m \approx 2\pi\hbar \times 1843 \text{ Hz}$ and $k_R = 2\pi/\sqrt{2}\lambda_R$, respectively, where $\lambda_R = 789.1 \text{ nm}$.

Using an adiabatic loading procedure, the BEC is initially prepared such that it occupies the lowest minimum of the lower SOC band shown schematically in Fig. 1(a). By suddenly switching off one of the two dipole trap beams, the condensate is allowed to spread out along the x -axis. After various expansion times, the BEC is imaged in-situ (see Fig. 2 and [27]), or after 13 ms of free expansion without a trapping potential or SOC (see Fig. 1b). In the negative x -direction, the BEC encounters an essentially parabolic dispersion, while in the positive x -direction, it enters a negative-mass region. This leads to a marked asymmetry in the expansion.

Experimental results together with matching numerical simulations are presented in Fig. 2. Three sets of data are presented with Raman coupling strength $\Omega = 2.5E_R$ and Raman detunings $\delta \in \{2.71E_R, 1.36E_R, 0.54E_R\}$. With decreasing δ , the dispersion relation (shown in the inset in the second row) develops a more pronounced double-well structure in the lower band. The BEC is initially placed at the global minimum of the lower band, and the expansion dynamics are initiated at time $t = 0$. Integrated cross sections are shown in the first row, the location of the 20% and 80% quantiles in second row, and the experimental and numerical time-slice plots in the third and fourth rows respectively. For $\delta = 2.71E_R$, the expansion is almost symmetric. For $\delta = 1.36E_R$, one sees a noticeable slowing down of the positive edge of the cloud. In the corresponding simulations of the Gross-Pitaevskii equation (GPE) this occurs at about 11 ms where the quasimomenta at about $40 \mu\text{m}$ enter the region of negative effective mass. Here one sees a pileup of the density and a dynamic instability, i.e. an exponential growth in the amplitude of phonon modes. For the smallest detuning $\delta = 0.54E_R$, the effect is even more pronounced, and in the corresponding GPE simulations, the pileup and instability start sooner at about 8 ms and $25 \mu\text{m}$.

At the lowest Raman detuning ($\delta = 0.54E_R$), the experiments show a slowdown at approximately 15 ms in the negative edge, which is not seen in the GPE simulations. We speculate that this might be due to finite temperature allowing a population in the second local minimum.

The dynamics of this system are governed by a coupled

set of GPEs describing the two spin components:

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix} = \begin{pmatrix} \frac{\hat{p}^2}{2m} + V_\uparrow & \frac{\Omega}{2} e^{2ik_R x} \\ \frac{\Omega}{2} e^{-2ik_R x} & \frac{\hat{p}^2}{2m} + V_\downarrow \end{pmatrix} \cdot \begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix}, \quad (1a)$$

$$V_{\uparrow/\downarrow} = -\mu \pm \frac{\delta}{2} + g_{\uparrow\uparrow/\uparrow\downarrow} n_\uparrow + g_{\uparrow\downarrow/\downarrow\downarrow} n_\downarrow \quad (1b)$$

where $\hat{p} = -i\hbar \vec{\nabla}$ is the momentum operator, μ is a common chemical potential, k_R is the Raman wave-vector, and $g_{ab} = 4\pi\hbar^2 a_{ab}/m$, where a_{ab} are the S -wave scattering lengths. For the $|\uparrow\rangle = |1, -1\rangle$ and $|\downarrow\rangle = |1, 0\rangle$ hyperfine states of ^{87}Rb , the three scattering lengths are almost equal and for our numerics we take $a_{\uparrow\uparrow} = a_{\uparrow\downarrow} = a_{\downarrow\downarrow} = a_s$. We compare our experimental results with 3D axially symmetric simulations for $\omega_\perp = 2\pi \times 162 \text{ Hz}$ and realistic experimental parameters. We use a discrete variable representation (DVR) basis (see e.g. [28]) with 4096×128 lattice points in a periodic tube of length $276 \mu\text{m}$ and radius $10.8 \mu\text{m}$.

One of the main theoretical results we wish to convey is that in many cases, a single-band effective Hamiltonian can capture the essential dynamics of a SOC BEC:

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = (E_-(\hat{p}) + gn + V_{\text{ext}}(x)) |\psi\rangle \quad (2)$$

where $E_-(\hat{p})$ is the dispersion of the lower band obtained by diagonalizing (1) for homogeneous states. For inhomogeneous densities this picture is locally valid for slowly varying densities, similar to the Thomas-Fermi approximation, and remains valid as long as the system is gently excited compared to the band separation, which is proportional to the strength Ω of the Raman coupling. With our parameters, the single-band model exhibits almost identical results to the multi-band description, quantitatively reproducing many aspects of the experiment. The approximate equality of the coupling constants allows one to define a spin-quasimomentum mapping that relates the two-component spin populations n_\uparrow and n_\downarrow to the quasimomentum q of the single-component state:

$$\frac{n_\downarrow - n_\uparrow}{n_\downarrow + n_\uparrow} = \frac{k - d}{\sqrt{(k - d)^2 + w^2}}, \quad (3)$$

where we have defined the dimensionless parameters $k = p/\hbar k_R$, $d = \delta/4E_R$, and $w = \Omega/4E_R$. This greatly simplified analysis shows that all of the interesting phenomena observed in the experiment – asymmetric expansion, the pileup, slowing down, and instabilities – follow from the modified dispersion relationship.

Having introduced the single-component theory, we now derive the hydrodynamics of this model by effecting a Madelung transformation $\psi = \sqrt{n}e^{i\phi}$ where $n(\vec{x}, t) = n_\uparrow + n_\downarrow$ is the total density at position \vec{x} and time t , and the phase $\phi(\vec{x}, t)$ acts as a quasimomentum poten-

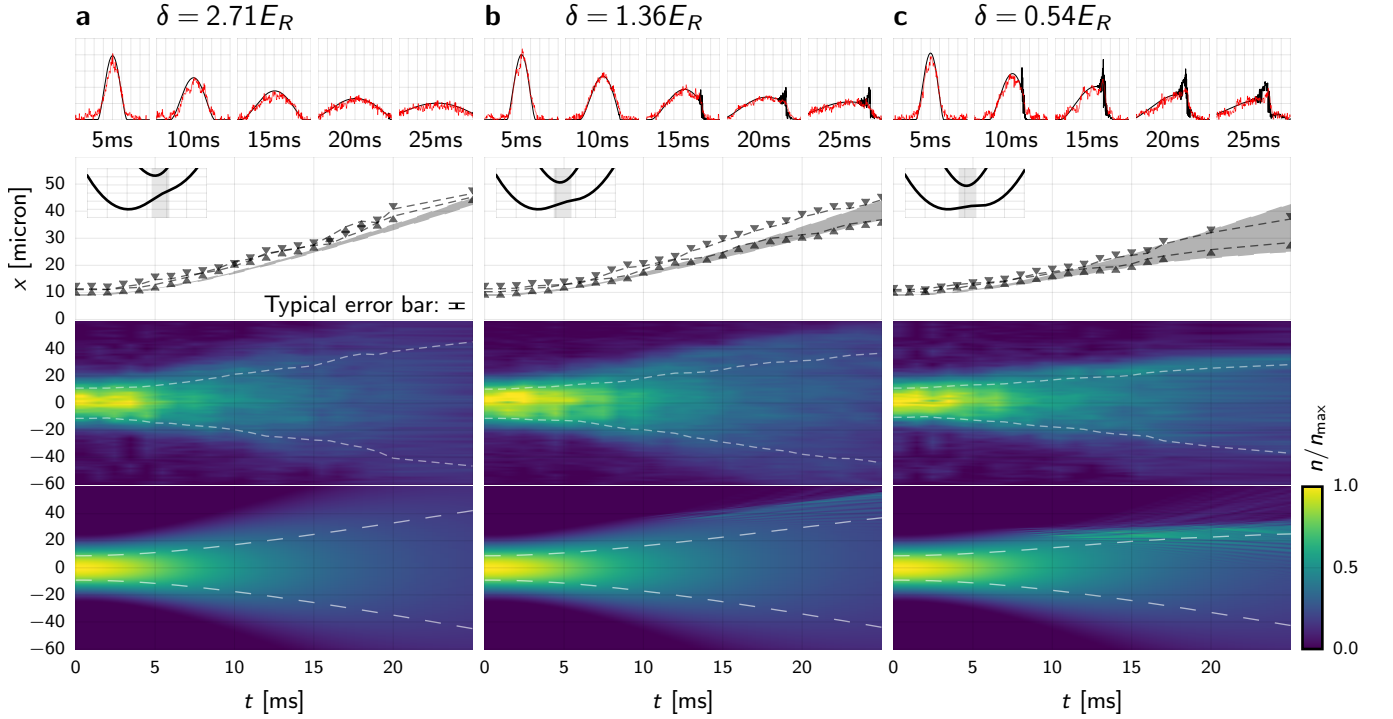


FIG. 2. Anisotropic expansion of a BEC along the direction of spin-orbit coupling (x -axis) with the Raman coupling strength $\Omega = 2.5E_R$, and various detunings of (a) $\delta = 2.71E_R$, (b) $\delta = 1.36E_R$, and (c) $\delta = 0.54E_R$, from left to right respectively. The first row shows the experimental integrated cross section of the condensate (dotted red curves) overlaid with results from the GPE simulation (solid black curves). The second row plots the location of the 20% and 80% quantiles with respect to the initial cloud center. The shaded regions present the GPE simulations, while the data points (and dotted lines to guide the eye) are the experimental measurements smoothed slightly by averaging the nearest three times. Error estimates are the standard deviation of 5 samples and are comparable with about twice the camera pixel resolution. The insets show the dispersion relation, with the region of negative mass lightly shaded. The bottom two rows show the evolution of the expanding condensate from the experiment (upper) and corresponding single-band axially-symmetric 3D GPE simulations (lower). The dashed white lines depict the quantiles from the plot above. All experimental data presented here are from in-situ imaging.

tial $\vec{p} = \hbar \vec{\nabla} \phi$. The hydrodynamic equations are

$$\frac{\partial}{\partial t} n + \vec{\nabla} \cdot (n \vec{v}) = 0, \quad (4a)$$

$$\frac{\partial \vec{v}_*}{\partial t} + (\vec{v}_* \cdot \vec{\nabla}) \vec{v}_* = \mathbf{M}_*^{-1} \cdot \overbrace{(-\vec{\nabla}[V_{\text{eff}} + V_Q])}^{\vec{F}}, \quad (4b)$$

$$[\mathbf{M}_*^{-1}]_{ij} = \frac{\partial E_-(\vec{p})}{\partial p_i \partial p_j}, \quad [\vec{v}_*]_i = \frac{\partial E_-(\vec{p})}{\partial p_i}, \quad (4c)$$

where \vec{v} is the group velocity and $\vec{j} = \vec{v}n$ is the current density. $V_{\text{eff}} = V_{\text{ext}}(\vec{x}, t) + gn(\vec{x}, t)$ is the effective potential, including both the external potential and mean-field effects. What differs from the usual Madelung equations is that third and higher derivatives of the dispersion $E_-(\vec{p})$ affect the velocity \vec{v}_* and quantum potential $V_Q(n, \vec{p})$. While for homogeneous matter, $\vec{v} = \vec{v}_*$, this relationship is broken in inhomogeneous matter and the quantum potential acquires terms beyond the usual quantum pressure term $V_Q(n) \propto \nabla^2 \sqrt{n}/\sqrt{n}$. For approximately homogeneous sections of the cloud, however,

these corrections are small: $\vec{v} \approx \vec{v}_*$ and the usual hydrodynamic behavior is realized. In particular, $\vec{\ddot{v}} \approx \mathbf{M}_*^{-1} \cdot \vec{F}$, so that the group velocity responds classically, accelerating against the force \vec{F} if the effective mass is negative. The experiment may be qualitatively explained using a Thomas-Fermi-like approximation where each point of the cloud is locally described by a plane-wave with local quasimomentum p . Initially, equilibrium is established between the external trapping force $-\nabla V_{\text{ext}}$ and the internal mean-field pressure $-\nabla(gn)$, with the quasimomentum $p = p_0$ minimizing the kinetic energy $E'_-(p_0) = 0$. About this minimum, the effective mass is positive, so once the trapping potential V_{ext} is reduced, the cloud starts to expand due to the mean-field pressure, generating an outward group velocity and quasimomenta. As the quasimomentum along the positive x -axis approaches the region of negative mass (see Fig. 3), the acceleration slows significantly compared with the acceleration along the negative axis, leading to the asymmetric expansion seen in Fig. 2: a manifestation of the broken

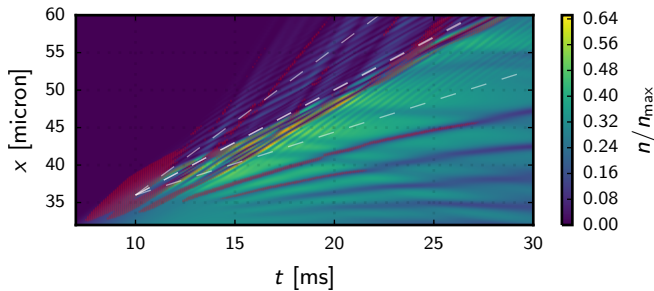


FIG. 3. Zoomed-in view of a 1D simulation matching the lower middle frame of Fig. 2 showing the total density $n = n_{\uparrow} + n_{\downarrow}$ as a function of time in the region where the dynamic instability first appears. The dashed lines are the three group velocities $v_g = E'_-(k)$ at the quasimomenta k where the effective mass $m_*^{-1} = E''(k)$ starts becoming negative (steepest line), the point of minimum negative effective mass (middle), and the point where the mass returns to positive (least steep line). Red points demonstrate where the local quasimomentum lies in the region of negative effective mass.

Galilean covariance and parity in SOC systems. Once the quasimomentum enters the region of negative mass, the acceleration opposes the force, and the cloud experiences the “self-trapping” effect where the positive mean-field pressure tends to prevent further expansion.

A similar effect has been reported in optical lattices near the edge of the Brillouin zone [22, 24, 29]. These “self-trapping” effects in lattices have been attributed to several different phenomena. One, based on a Josephson effect with suppressed tunneling between neighboring sites [22, 30], was predicted from a variational framework [31]. Another explanation is that the sharp boundary is a “gap soliton” [32], though this explanation is disputed by [30] on the basis that solitons should remain stable whereas the latter observe self-trapping only for a finite period of time. Finally, self-trapping has been explained in terms of the Peierls-Nabarro energy barrier [33]. In all of these cases, the self-trapping occurs where the effective mass becomes negative, but the interpretation of the self-trapping effect in optical lattices is complicated by the presence of spatial modulations in the potential.

The beauty of engineering dispersions with SOC BECs is that lattice complications are removed. The success of the single-band model in reproducing the experiment demonstrates clearly that the self-trapping results from the effective dispersion relationship. A single-band model using the dispersion of the lowest band in an optical lattice is able to explain the previous observation of self-trapping in lattice systems, clearly demonstrating the importance of the band structure and de-emphasizing the role of the underlying lattice geometry of coupled wells. This result is confirmed by using a tight-binding approximation to map the optical lattice of [30] to a single band model, which reproduces their results. In our

simulations, although the boundary appears to be very stable, it is “leaky”. This can be seen from Fig. 3 where the boundary maintains its shape, but permits a small number of fast moving particles to escape. Similarly, in optical lattice systems such fast moving particles are responsible for the continued increase in the width of the cloud seen by [30] even though the boundary remains stopped. This may resolve the apparent discrepancy between [30] and [32] with a quasi-stable but leaky “gap soliton”.

What sets the limiting velocity of the expanding edge? In the optical lattice system of Ref. [24], the limiting velocity was observed to lie at the inflection point where the mass first starts to become negative. In contrast, our GPE simulations for SOC BECs clearly show this limiting velocity to lie fully inside the negative mass region, near the point of maximum negative acceleration (maximum negative inverse mass) (see Fig. 3). While this qualitatively describes the limiting velocity, the full effect is somewhat subtle. The limiting velocity ultimately depends on several factors, including the preparation of the system [34]. It requires a quasi-stable boundary which is tied to the negative mass through a dynamical instability. From the GPE simulations, the picture emerges that once the cloud enters the negative-mass region, small fluctuations grow exponentially forming the sharp boundary of the cloud. Initially these growing modes appear chaotic, but as is typical with dispersive shock-waves (see [8] and references therein), energy is “radiated” from the boundary in the form of phonons and soliton trains. These trains are clearly visible in Fig. 3. As energy is dissipated, the boundary appears to sharpen due to non-linear effects. This seems critically connected to the negative mass as a similar boundary with positive mass dispersion broadens [34].

In conclusion, we have studied negative-mass hydrodynamics both experimentally and theoretically in an expanding spin-orbit coupled Bose-Einstein condensate. The experimental results are quantitatively reproduced with an effective single-band GPE-like model derived from the SOC Hamiltonian. With this model, we see that the pileup and subsequent boundary behavior are intimately related to the presence of a negative effective mass $m_*^{-1} = E''(p)$ in the effective dispersion for the band. From linear response theory, one finds a dynamical instability closely associated with the negative mass that leads to the sudden increase in density. The boundary clearly demonstrates radiation of phonons and soliton trains, which appear to remove energy from the region, thereby allowing the boundary to stabilize. The stability of the boundary and its final velocity depend critically on the existence of a negative mass. With this work, we have also clarified the interpretation of self-trapping phenomena observed in optical lattices [22, 30], demonstrating that this is naturally explained by a negative effective mass. Spin-orbit coupling provides a powerful tool

for engineering the dispersion relationship $E_-(p)$ without the additional complications of spatial modulations that appear in the context of optical lattices.

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NUMERICAL SIMULATIONS

In principle, since the trap is highly elongated, one can use an effective 1D simulation by tuning the coupling constants appropriately as described in [35, 36]. For the experimental parameters, this effective 1D approximation works quite well, but since there are some radial excitations, we compare directly with 3D axially symmetric simulation with $\omega_{\perp} = 2\pi \times 162$ Hz. Our simulations are performed using an axially symmetric DVR basis (see e.g. [28]) with 4096×128 lattice points in a periodic tube of length $276 \mu\text{m}$ and radius $10.8 \mu\text{m}$.

In Fig. 4 we compare the single-band model (2) to the full two-component model (1), demonstrating that for the experiment under consideration, all the relevant phenomena result purely from the modified dispersion relationship $E_{-}(\hat{\mathbf{p}})$. Even when studying systems with significant population of the upper band such as the dynamical spin-density waves demonstrated in [37], we find that the single-component model (2) captures most of the bulk effects: i.e. it correctly models the dynamical behavior of the total density $n_{\uparrow} + n_{\downarrow}$, but the significant population of the upper branch breaks the spin-quasimomentum mapping (3). In this figure we also compare 3D axially-symmetric simulations with 1D simulation where we have

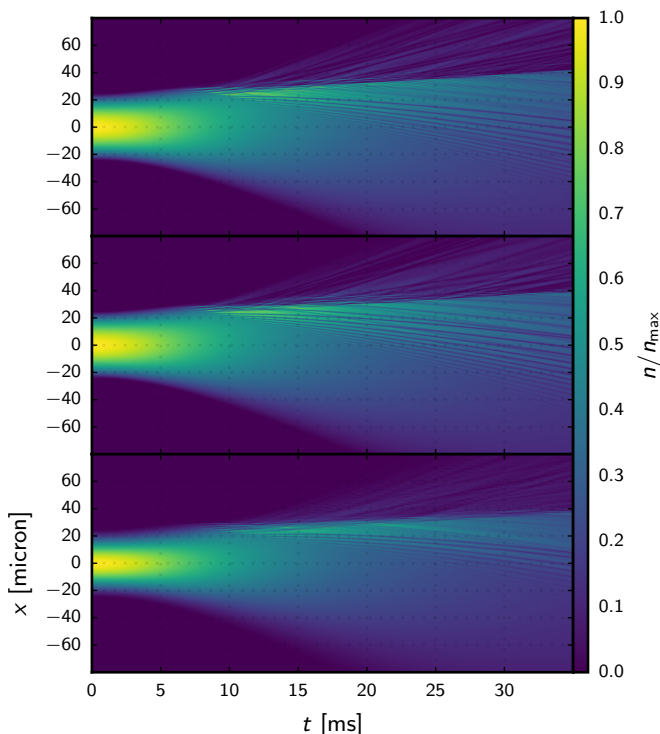


FIG. 4. **Comparison between simulations.** Comparison of the full two-component 1D GPE model (1) (top) with the 1D single-band model (2) (middle) and the axially symmetric 3D single-band model (bottom) for detuning $\delta = 0.54E_R$ and Raman coupling strength $\Omega = 2.5E_R$.

tuned the coupling constant using the relationship in [35]. This comparison demonstrates that, while the 1D simulations qualitatively match the experiments, they exhibit some quantitative differences with the 3D simulations.

EXPERIMENTAL METHODS

The experiment begins with approximately 10^5 ^{87}Rb atoms confined by two trapping dipole beams which together produce harmonic trapping frequencies of $(\omega_x, \omega_y, \omega_z) = 2\pi \times (26, 170 \text{ and } 154) \text{ Hz}$. SOC is then generated along the x -axis by two perpendicular Raman beams of wavelength $\lambda_R = 789.1 \text{ nm}$ (see Fig. 5a). The Raman beams coherently couple the $|F, m_F\rangle = |1, -1\rangle$ and $|1, 0\rangle$ states. A homogeneous bias field of $B \approx 10 \text{ G}$ is applied along the x -axis, leading to a Zeeman splitting of the atomic levels. The quadratic Zeeman shift of $7.8E_R$ effectively decouples the $|1, +1\rangle$ state from the Raman dressing, resulting in a system with pseudo-spin $\frac{1}{2}$ where we identify $|\uparrow\rangle = |1, -1\rangle$ and $|\downarrow\rangle = |1, 0\rangle$ (see Fig. 5b). After the atoms are dressed by the Raman beams with a specific Raman coupling strength Ω and detuning δ , the vertical dipole beam is rapidly switched off, reducing the confinement along the x -axis from $\omega_x = 2\pi \times 26 \text{ Hz}$ to $\omega_x = 2\pi \times 1.4 \text{ Hz}$. This allows the SOC BEC to expand along the direction of the spin-orbit coupling, revealing the rich dynamics discussed in the main text. The extent of the cloud as depicted in Fig. 2 is taken from in-trap images. Figure 1b depicts experimental images taken after an in-trap expansion time denoted to the left of the image, followed by 13 ms free expansion.

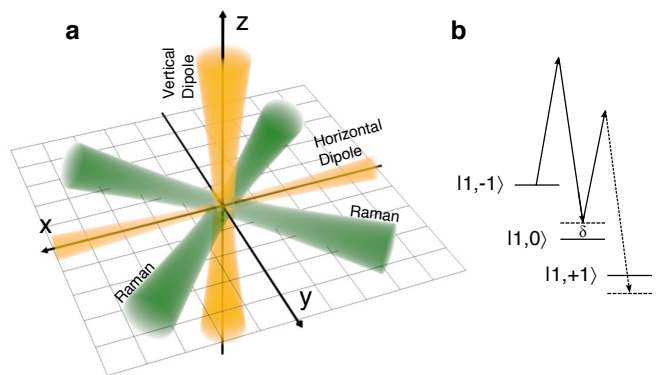


FIG. 5. **a**, Experimental arrangement of the trap (yellow) and the Raman (green) beams. The angle between the Raman beams is 90° . **b**, Raman coupling scheme in the $F = 1$ manifold. The $|1, +1\rangle$ state is effectively decoupled due to the quadratic Zeeman shift induced by an external bias field along the x -direction.

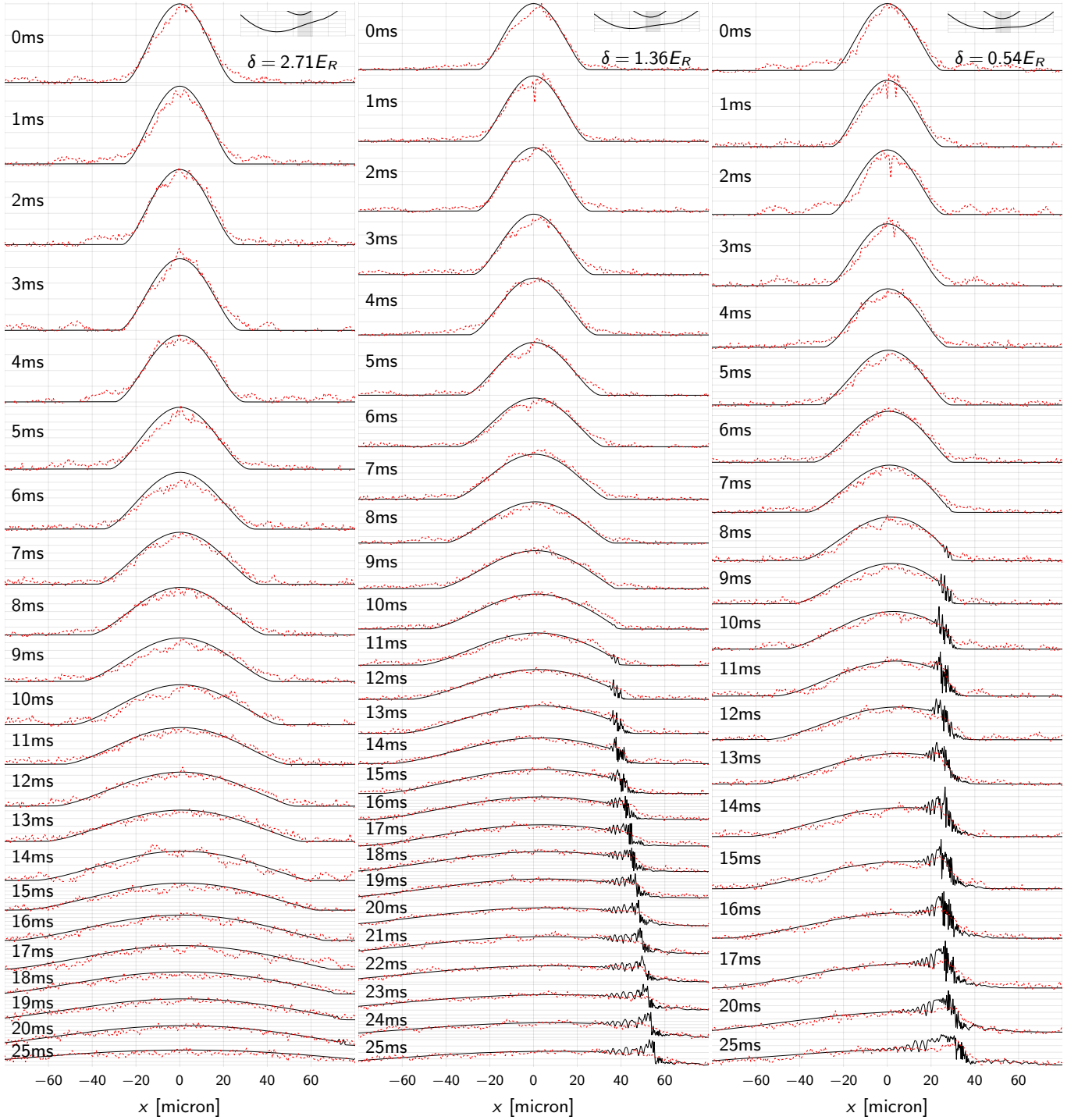


FIG. 6. **In-situ data: Experiment and Theory.** In-situ experimental integrated cross section of the condensate (dotted red curves) overlaid with results from the axially symmetric single-band 3D GPE simulations (solid black curves) with the Raman coupling strength $\Omega = 2.5E_R$, and various detunings of (a) $\delta = 2.71E_R$, (b) $\delta = 1.36E_R$, and (c) $\delta = 0.54E_R$, from left to right respectively. The top insets show the dispersion relation, with the region of negative effective mass lightly shaded.

COMPARISON

To supplement the comparison between the numerical and experimental results, we include in Fig. 6 a high-

resolution comparison between the numerical and experimental in-situ images. Note that we have not added noise or pixel averaging effects to the numerical simulations so that underlying features (solitons etc.) remain unobscured.